Alois Lechicki
[Version 1.2; last revised: April 14, 2026]

Does Classical Logic Imply a Risk-Free World?

Summary

This note examines the assertion that classical logic implies a risk-free world. We demonstrate that this perspective confuses truth with knowledge. While classical logic attributes definite truth values to all propositions, these values are not necessarily known to an agent. Consequently, uncertainty – and therefore risk – persists. Adopting R. T. Cox's interpretation of probability as an extension of logic, we conclude that risk arises from incomplete information, not any limitation of classical logic.

0. Introduction

In this note, we will discuss the claim that:

Logic in the real world is not classical logic – otherwise we would have a risk-free reality.

At first glance, this statement may seem plausible: if classical logic assigns a definite truth value (true or false) to every proposition, then one might suspect that uncertainty, and hence risk, should not arise in a world governed by such logic. From this perspective, the existence of risk would suggest a limitation of classical logic, and perhaps an alternative 'non-classical' logical system is needed.

The purpose of this note is to clarify this intuition and demonstrate that it is misleading. The key point is that classical logic is a theory of truth, not a theory of knowledge. While classical logic asserts that every well-formed proposition has a definite truth value, it does not provide agents with access to that value. In particular, it does not eliminate uncertainty about which propositions are true. However, risk arises precisely from such uncertainty.

To make this distinction precise, we briefly recall how uncertainty is treated in a mathematical framework. Building on the work of R. T. Cox ([C1], [C2] and [C3]), probability theory can be viewed as a consistent extension of classical logic to situations in which truth values are unknown. In this sense, probability theory does not replace classical logic, but rather complements it by providing a calculus for reasoning under incomplete information.

The central question we address can therefore be formulated as follows:

Does classical logic imply a risk-free world?

We will argue that the answer is NO. Even in a world in which all propositions obey classical logic, uncertainty – and hence risk – naturally arises from the limited information available to individuals. A risk-free world would require not a different logic, but rather complete knowledge.

1. Classical Logic: What It Actually Says

Let us briefly recall the basic framework of classical propositional logic to clarify terminology and avoid a common source of confusion.

Let $\mathcal{L}$ be a prepositional language and let $A, B, \ldots \in \mathcal{L}$ denote propositions. By 'propositional language', we mean a formal system comprising propositional variables and logical connectives, along with inductively defined rules for constructing well-defined formulas ([P1]).

In classical logic, each proposition is assigned a *truth value* in the set $\{0,1\}$, where 1 stands for *true* and 0 for *false*. A (classical) valuation is a map

$$v: \mathcal{L} \to \{0,1\}$$

that respects the usual truth-functional definitions of the logical connectives. The fundamental laws of classical logic, including the laws of *non-contradiction* and *excluded middle*, follow immediately from these definitions. Thus, in the classical setting, every proposition A is assigned a definite truth value of either 1 or 0. There are no intermediate or indeterminate values.

It is important to stress a distinction that is often left implicit at this point. Classical logic is a *theory of truth conditions;* it specifies how the truth value of complex propositions depends on the truth values of their constituents. However, it does not address the question of how these truth values are determined, nor whether they are known to a given agent.

In particular, classical logic asserts that $v(A)$ belongs to the set $\{0,1\}$ for a given proposition A, but does not provide any mechanism by which an agent can determine whether $v(A)$ equals 1 or 0. The existence of a truth value should therefore not be confused with its epistemic accessibility.

This observation is central to what follows. The presence of a well-defined truth value for every proposition does not preclude uncertainty on the part of an observer. Classical logic leaves open the possibility that the truth value of many propositions of interest is unknown or can only be inferred partially from available information.

In summary, classical logic imposes a binary structure on propositions but remains silent about the knowledge state of an agent reasoning about them. It is precisely this gap between truth and knowledge that gives rise to notions of uncertainty – and ultimately risk.

2. Where Risk Comes From

We will now move on to discuss the origin of risk and its relation to the logical framework described above. The central point is that risk does not arise from the structure of logic itself, but from the limited information available to an agent reasoning about the world.

Let A be a proposition of interest, for example, "*an insurance claim occurs within the next year.*" In the classical setting, A has a definite truth value $v(A) = \{0, 1\}$. However, in typical applications, this value is not known to the observer at the time a decision must be made. Therefore, the agent is faced with a situation where the truth of A is uncertain. This results in a fundamental distinction:

- *Logical truth* refers to the objective truth value $v(A)$ assigned by a valuation.
- *Epistemic state* refers to the information available to an agent concerning A.

Classical logic governs the former, but is silent about the latter. In particular, it does not provide a mechanism for determining $v(A)$ from partial or incomplete information.

In the context of risk theory, one is typically concerned with propositions describing future or otherwise unobserved events. Let A denote such an event and suppose that decisions must be made prior to the disclosure of its truth value. The agent must therefore act under uncertainty, which can be described as the absence of complete knowledge about $v(A)$.

Risk arises when this uncertainty is coupled with consequences. More precisely, consider a decision whose outcome depends on the truth value of A. Since $v(A)$ is unknown at the time of decision, the agent is exposed to variability in outcomes. This variability, together with the associated gains or losses, constitutes risk. In other words, risk emerges from uncertainty about the realisation of events and their consequences, that is, from incomplete information about propositions that nevertheless possess definite truth values (cf. [K1] and [J1]).

It is important to emphasise that this phenomenon is entirely compatible with classical logic. Even if every proposition possesses a well-defined truth value, uncertainty persists whenever that value is not accessible to the agent. Therefore, the presence of risk does not necessarily indicate a failure or inadequacy of classical logic. Instead, it reflects the inherent limitations of practical decision-making processes.

In summary, risk originates from the gap between truth and knowledge. In classical logic, propositions are either true or false. However, this approach does not eliminate uncertainty about which of these alternatives applies. This discrepancy is precisely what necessitates a mathematical treatment of uncertainty, to which we turn next.

3. From Logic to Probability

As discussed in the previous sections, classical logic provides a calculus of truth, but does not address the problem of reasoning under incomplete information. In such situations, it is common to look for ways to extend the logical framework used to represent and manipulate degrees of plausibility for propositions whose truth values are unknown.

R. T. Cox proposed a systematic solution to this problem ([C1]). The central idea is to generalise the binary valuation $v(A) \in \{0,1\}$ to a function that assigns to each proposition A, given background information B, a real number

$$P(A \mid B) \in [0,1].$$

This is interpreted as the *degree of belief* (or *plausibility*) of A conditional on B. This assignment is required to satisfy certain natural consistency conditions. In particular:

- **Qualitative agreement with logic**: If A logically implies B, then $P(A \mid C) \leq P(B \mid C)$.
- **Consistency under equivalent representations**: Logically equivalent propositions must be assigned the same degree of belief.
- **Functional dependence**: The plausibility of compound propositions, such as $A \wedge B$, should depend only on the plausibilities of their constituents in a consistent manner.

Cox showed that, under mild regularity assumptions, any such system of plausible reasoning is (up to a monotone transformation) equivalent to the standard calculus of probability. In particular, the following rules must hold:

- the product rule

$$P(A \wedge B \mid C) = P(A \mid B \wedge C)P(B \mid C),$$

- the sum rule

$$P(A \mid C) + P(\sim A \mid C) = 1.$$

These are precisely the defining relations of conditional probability. Therefore, probability theory does not emerge as an independent formalism, but rather as a unique extension of classical logic to situations involving uncertainty.

This perspective naturally interprets probability as a measure of rational belief rather than merely as long-run frequency. In particular, the quantity $P(A \mid B)$ represents the degree to which an agent possessing information B should regard proposition A as plausible, provided that it is logically consistent.

In summary, while classical logic operates with exact truth values, probability theory extends this framework by introducing a continuous scale of plausibility, enabling coherent reasoning in the presence of incomplete information. This extension is not arbitrary, but is dictated by the same principles of consistency that underlie classical logic itself.

4. Derivation of Bayes' Rule

The fundamental formula for updating beliefs in light of new information can be derived from the basic rules above.

Let A, B and C be propositions. From the product rule, we have

$$P(\, A \wedge B \mid C\,) = P(\, A \mid B \wedge C\,)P(\, B \mid C\,)$$

and

$$P(\, A \wedge B \mid C\,) = P(\, B \mid A \wedge C\,)P(\, A \mid C\,),$$

because conjunction is commutative: $A \wedge B = B \wedge A$. Hence

$$P(\, A \mid B \wedge C\,)P(\, B \mid C\,) = P(\, B \mid A \wedge C\,)P(\, A \mid C\,)$$

and assuming $P(\, B \mid C\,) > 0$ we get

$$P(\, A \mid B \wedge C\,) = \frac{P(\, B|A \wedge C\,)P(\, A|C\,)}{P(\, B|C\,)}.$$

This Bayes' rule provides a systematic method for updating the plausibility of a proposition A in light of new information B, given background information C. The commonly used form

$$P(\, A \mid B\,) = \frac{P(\, B|A\,)P(A)}{P(B)}$$

is obtained by fixing background information C and suppressing it in the notation, writing $P(A)$ in place of $P(\, A \mid C\,)$.

Bayes' rule expresses the fundamental principle that prior assessments are revised in light of new evidence in a way that remains consistent with the underlying rules of probability. Note that this rule does not represent an additional assumption, but is a direct consequence of the basic structure of the probability calculus derived in the preceding section. In particular, Bayes' rule provides the mathematical mechanism by which uncertainty arising from incomplete knowledge of truth values can be updated as new information becomes available. Consequently, it plays a central role in the quantitative treatment of risk.

5. Interpretation of Probability

The formal development in the preceding sections shows how probability arises as a consistent extension of classical logic. This raises the question of how the quantity $P(\, A \mid B\,)$ should be interpreted. Let us discuss two main interpretations of probability that are commonly distinguished in the literature.

- **Frequentist interpretation**. In this view, the probability of an event is defined in terms of long-run relative frequencies. More precisely, if an experiment is repeated under identical conditions, the probability of event A is identified with the limiting frequency with which A occurs.

While this interpretation is appropriate in many applications, it is inherently tied to repeatable experiments and cannot be applied directly to propositions referring to single events or unique situations. Statements such as 'a given driver will incur a claim next year' cannot be naturally interpreted in this way without introducing an artificial reference class.

- **Epistemic (Bayesian) interpretation** ([1]). An alternative viewpoint, closely related to the considerations of R. T. Cox, interprets probability as a measure of degree of belief. In this

[1] The epistemic (Bayesian) interpretation of probability is closely related to the subjective probability theory developed by Bruno de Finetti, yet the two are not identical. While they share the same mathematical framework, they differ in their philosophical foundations: the Bayesian approach is grounded in principles of logical consistency, whereas de Finetti's formulation is based on a behavioural interpretation in terms of coherent betting.

framework, the quantity $P(A \mid B)$ represents the plausibility of the proposition A given the information B, subject to the requirement of logical consistency. From this perspective, probability quantifies uncertainty arising from incomplete information. The underlying propositions are still assumed to possess definite truth values, in accordance with classical logic; probability reflects only the epistemic state of an agent who does not know these values.

This interpretation has the advantage of applying uniformly to a wide range of situations, including those involving single events, partial information, or evolving knowledge. Moreover, as discussed in Section 3, the rules governing these degrees of belief are not arbitrary but follow from general consistency requirements.

- **Rational belief, disagreement, and the limits of convergence.** It is important to emphasise that interpreting probability as a degree of rational belief does not mean that different people must give the same proposition the same probability. Even when presented with the same explicit evidence, domain experts may reach different conclusions, reflecting differences in background knowledge, prior assumptions or modelling choices. In the Coxian framework, rationality is defined as internal consistency; degrees of belief must satisfy the axioms of probability and respect the logical relations between propositions. However, these requirements do not uniquely determine numerical values. Therefore, rational belief should be understood as coherent belief rather than universally agreed belief. Disagreement between agents is therefore fully compatible with rationality, provided each agent's system of beliefs is internally consistent and updated in accordance with the principles of probability theory.

A striking example of diverging expert assessments is provided by the analysis of the Space Shuttle Challenger disaster. Following the disaster, Richard Feynman observed that NASA management had estimated the probability of a catastrophic O-ring failure to be around 1 in 100.000, whereas the engineers involved in the project had assessed the risk to be closer to 1 in 100 ([F1]). Despite having access to broadly similar technical data, this dramatic discrepancy – spanning several orders of magnitude – occurred due to differences in interpretation, modelling assumptions and institutional context, highlighting how these factors can lead to profoundly different probabilistic assessments.

At the same time, Bayesian rationality should be regarded as a normative ideal rather than a descriptive model of actual human behaviour. It specifies how agents ought to reason rather than how they actually do reason. In particular, the convergence of beliefs, which is often emphasised in Bayesian theory, is a normative result that depends on strong assumptions. These include the requirement that agents consistently update their beliefs according to Bayes' rule, share a common probabilistic model, and are primarily concerned with truth-seeking. In practical settings, however, these assumptions are often violated. For example, experts may differ in their interpretation of evidence, adopt different modelling frameworks, or rely on distinct bodies of background knowledge. Furthermore, in certain contexts, agents may act strategically due to institutional or political incentives. In such cases, the probabilities they report may not reflect purely epistemic beliefs, so convergence cannot be expected. More generally, when agents have hidden agendas, their stated probabilities may reflect strategic considerations rather than rational belief.

- **Relation to risk theory**. For the purposes of risk theory, the Bayesian interpretation is particularly appropriate. Decisions are usually made based on incomplete information about future events, and probabilities offer a consistent framework for representing and updating this information. However, it is important to stress that using probabilities does not change the underlying logical structure of propositions. Each proposition remains either true or false; probability merely quantifies the uncertainty surrounding which alternative is true.

Among the various interpretations of probability, the Bayesian (subjective) view – associated with Bruno de Finetti – is especially well suited to this context, as it interprets probabilities as coherent degrees of belief about uncertain events and allows the incorporation of both data and expert judgement.

By contrast, the frequentist interpretation, which defines probability as a long-run frequency, is primarily adapted to repeatable experiments and large samples, and is therefore less natural for unique or forward-looking risk assessments. In practice, risk management often combines both perspectives: frequentist methods are used for statistical estimation, while Bayesian reasoning provides the conceptual foundation for interpreting probabilities and supporting decisions under uncertainty. However, note that to apply the frequentist interpretation, the case in question must be placed into a suitable reference class (i.e. a group of 'similar' cases) and probability defined as the relative frequency within that class. The problem is that there is often no unique, objectively correct reference class. This indicates that probabilities cannot be uniquely defined as long-run frequencies in many real-world risk situations, as it is unclear which group of 'similar cases' should be used for comparison.

Although mathematically important, abstract Kolmogorovian probability theory has limited direct application in risk management as it does not specify how probabilities should be interpreted, obtained, or updated in uncertain situations. Notably, the framework presupposes a given probability measure, yet it offers no guidance on how this measure should be constructed when dealing with incomplete information, expert judgement, or unique future events — all of which are central features of risk management. Consequently, while the Kolmogorov model ensures internal consistency, it does not address the fundamental practical issue of how probabilities should initially be assigned. For this reason, the Bayesian approach provides a more practical foundation for risk-based decision-making, even though it can be embedded within the Kolmogorovian formalism.

6. Why Classical Logic Does NOT Eliminate Risk

We are now in a position to address the central question posed in the introduction. The idea that a world governed by classical logic would be 'risk-free' is based on a misunderstanding of the role of logic in analysing uncertainty.

Let A be a proposition that describes an event of interest; for example, the occurrence of an insurance claim within a given time period. In classical logic, A has a definite truth value $v(A)$ in $\{0, 1\}$. However, this does not imply that the value $v(A)$ is known to an agent when a decision must be made. The crucial observation is that risk is a consequence of uncertainty about truth values, not their absence. Classical logic guarantees that each proposition is either true or false, but provides no information as to which alternative applies in a given situation. Consequently, an agent operating under incomplete information must assign degrees of belief to competing possibilities and make decisions accordingly.

To make this point explicit, consider a decision whose outcome depends on the truth value of a proposition A. Suppose the agent's available information is represented by B and the agent assigns a probability $P(A|B)$ reflecting the plausibility of A. Even though A is either true or false in reality, the agent faces uncertainty prior to observing its realisation. The consequences of the decision therefore depend on an unknown quantity, and this dependence gives rise to risk.

It follows that the presence of risk is entirely compatible with a classical, bivalent notion of truth. Eliminating risk would require the agent to have complete knowledge of the truth value of every relevant proposition A. In other words, a 'risk-free' world would be one in which uncertainty is absent because all relevant information is available. This condition is epistemic in nature and does not depend on the underlying logical system.

This reasoning also clarifies why the introduction of non-classical logics is unnecessary for accounting for risk. Although alternative logical systems may be useful for modelling vagueness, inconsistency or other phenomena, the existence of risk already follows from the combination of two features:

- propositions have definite truth values, and
- these values are not fully known to the agent.

In summary, classical logic does not eliminate risk because it does not eliminate ignorance. Risk arises from the gap between truth and knowledge, and this gap persists regardless of whether the underlying logic is classical.

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